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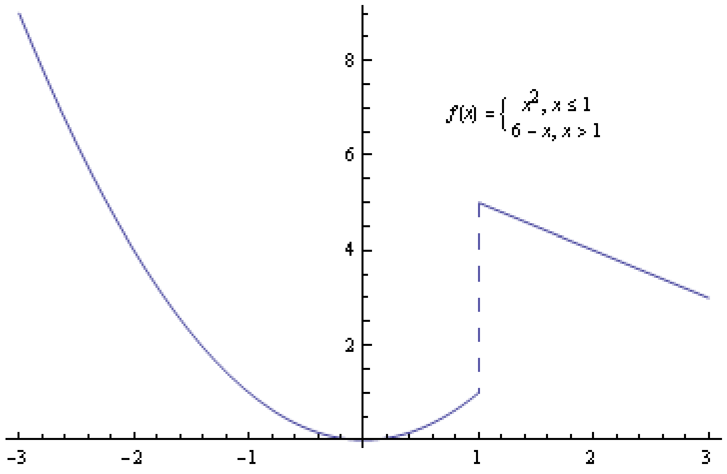
Discontinuity 1: Jump Discontinuities



$$\boxed{\lim_{x\to c^-} f(x) \neq \lim_{x\to c^+} f(x) \qquad \Delta l = |l_1 - l_2|}$$

Jump discontinuities are also called **simple discontinuities**, or **continuities of the first kind**.

Is given a **piecewise function** like this one

$$f(x) = \begin{cases} x^2, & x \le 1 \\ 6 - x, & x > 1 \end{cases}$$

We can consider it like two separate functions - one that is defined for x less than or equal to 1 and another function that is defined for x greater than 1. It is useful to think of it this way when we eventually use integration, differentiation, and other such mathematical tools.

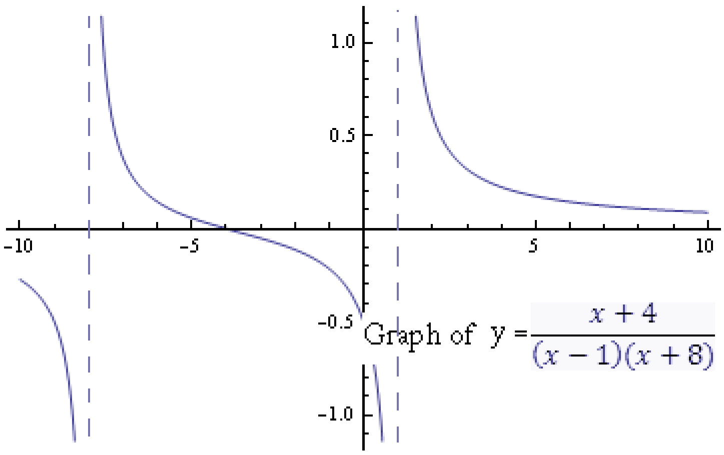
However, as it is written, f is a single function, since it is defined piece-by-piece. Note that the function doesn't fit to our definition of being continuous.

The two pieces have a different value at x = 1, and we can see in the graph that our function f seems to "jump" from one branch to the other. Note this this jump makes the function discontinuous. We refer to this as a **jump discontinuity**.

Notice that the function's discontinuity is entirely dependent on the value of the two branches of the function. Because of this, we can't just look at a piecewise function and immediately see if there is a jump discontinuity.

Jump discontinuities occur where the function approaches two different values from either side of the discontinuity. In our example, on the right side of x = 1, the function is approaching the value f(1) = 5. On the left side of x = 1, the function is approaching the value f(1) = 1. Thus, it has a jump discontinuity. Formally, we can check this by checking if the **left-hand limit** and the **right-hand limit** of the function correspond to the same value at a given point.

Discontinuity 2: Asymptotic Discontinuities



$$\boxed{\lim_{x \to c} f(x) = \pm\infty}$$

In the Function, $f(x) = \frac{x+4}{(x-1)(x+8)}$ we know that the domain is limited to all real numbers except 1 and -8.

Often, the most interesting points in a function are the problematic points, and indeed, we can see in the graph that the function behaves very strangely at the holes in the domain.

As we approach x = 1 we have:

The dotted lines represent **asymptotes**; they are values for which the function never takes a value, yet still approaches. The asymptotes we see with this function are called **vertical asymptotes** because they are vertical lines. There are also **horizontal asymptotes.**

In general, **asymptotes occur when a function approaches infinity at a specific value of x or y**. If a function has values on both sides of an asymptote, then it cannot be connected, so it must have a discontinuity at the asymptote. **We look for asymptotes at points where the denominator becomes zero.**

Discontinuity 3: Point Discontinuities

$$\boxed{\lim_{x \to c} f(x) \neq f(c) \quad \textit{or it doesn't exist}}$$

This point of discontinuities are also called **removable discontinuities**.

Sometimes we come across functions that are defined differently for a certain point. Consider the function $f(x) = \begin{cases} 1, & x = 3 \\ x^2, & \textit{all other real } x - \textit{values} \end{cases}$.

We defined the value of the function to be 1 at the point x = 3, yet, the rest of the function is dictated by $f(x) = x^2$. We can see in the graph that the function is continuous except for at x = 1. It is discontinuous at a single point, and this discontinuity is called a **point discontinuity**.

In general, **point discontinuities occur when a function is defined specifically for an isolated x-value.** However, this does not guarantee a point discontinuity. For example, if we change our function slightly to $f(x) = \begin{cases} 9, & x = 3 \\ x^2, & \textit{all other real } x - \textit{values} \end{cases}$ it becomes continuous. This is because we have defined the value of the function at f(3) precisely to be the value of the function $f(x) = x^2$ at x = 3.

Point Discontinuities also arise when our function has a denominator that can be equal to zero, but that part of the denominator can also be cancelled out with a like term in the numerator. Consider the function $f(x) = \frac{x^2(x-2)}{x-2}$. If we try to find the value of the function at x = 2, we end up getting $f(2) = \frac{2^2(2-2)}{2-2} = \frac{0}{0}$.

0/0 represents an undefined number - i.e. the function does not exist at that point. However, if we restrict the function to a domain that does not include x = 2, we can simply cancel out the $\frac{(x-2)}{(x-2)}$ and be left with $f(x) = x^2$. This leaves us to define the function as $f(x) = \frac{x^2(x-2)}{(x-2)} = \begin{cases} x^2, & x \neq 2 \\ undefined, & x = 2 \end{cases}$.

We have effectively removed the discontinuity to show that the function behaves exactly like $f(x) = x^2$, except at x = 2, where it is undefined.

In conclusion, **point discontinuities also occur when we can cancel a term in the denominator and the numerator**.